

Electric Motors and Drives

To calculate the peak power and torque produced by an electric motor, you will need to know the following:

- Motor supply voltage: $V_s [V]$
- Peak motor current: $I_s [amps]$
- Motor velocity: $N [rpm]$

The power produced by the motor can be calculated as:

$$P [\text{watts}] = V_s [\text{volts}] \times I_s [\text{amps}] \quad (\text{Eq. 1})$$

Converting power to units of horsepower:

$$P [\text{hp}] = P [\text{watts}] \times 0.001341 [\text{hp/watt}] \quad (\text{Eq. 2})$$

Since **power is equal to work divided by time**, we can use the definition of horsepower to understand the useful torque produced by the motor:

$$1 [\text{hp}] = 750 [W] = 550 [\text{lb-ft/s}] = 33,000 [\text{lb-ft/min}] \quad (\text{Eq. 3})$$

Now let's select a real motor to work with from the Motor Specifications link on the course website:

<http://www2.mae.ufl.edu/designlab/motors/Motor%20Specifications.pdf>

We will select one of the DC right angle drive gear motors used in the lab.

44 RPM 12/24 VDC 1/70 HP ENTSTORT GEARMOTOR



- 44 RPM at 12 VDC no load
- Voltage 12/24 DC
- Amps 1-1/2 full load
- Reversible
- Duty continuous
- Shaft 10 mm x 1 1/4" with 8 mm threaded end
- Mount 3 bolt on 2" B.C.
- Size 6 3/4" x 2 5/8" x 4"

From the basic equations presented above:

$$P = 12 \text{ V} \times 1.5 \text{ A} = 18 \text{ W} = \mathbf{0.024 \text{ hp}}$$

Comparing this value to the rated power of the motor ($1/70 = \mathbf{0.014 \text{ hp}}$), we see the actual power produced is substantially less than the computed electrical power. This loss is due to the electrical efficiency of the [brushed-type motor](#). The electrical efficiency will be denoted by η_{motor} and for the purpose of this course, we will assume the following:

$$\eta_{\text{motor}} \approx 60\%$$

Now the power can be more accurately computed as follows:

$$P = V \times I \times \eta_{\text{motor}} = 12 \text{ V} \times 1.5 \text{ A} \times 0.6 = 10.8 \text{ W} = 0.014 \text{ hp}$$

Using the definition of hp (Eq. 3):

$$0.014 \text{ hp} = \mathbf{7.9 \text{ lb-ft/s}} = \mathbf{95.0 \text{ lb-in/s}}$$

Therefore, this motor should be able to lift a 7.9 lb load at the rate of 1 foot per second; this is equivalent to lifting a 1 lb load at 7.9 feet per second.

Note the difference is you trade torque for speed or vice versa.

Looking at it in units of lb-in instead of lb-ft, this motor should be able to lift a 95 lb load at the rate of 1 inch per second; this is the same as lifting a 1 lb load at 95 inches per second.

Suppose we wish to calculate the velocity of our robot if we consider using these motors to power the drive wheels. First, we need to select a wheel diameter. Returning to parts found in lab, let's arbitrarily select an 8" diameter wheel to get a baseline. The linear velocity can simply be calculated using the circumference of the wheel:

$$V [\text{in}/\text{min}] = \pi \times D [\text{in}/\text{rev}] \times N [\text{rev}/\text{min}] \quad (\text{Eq. 4})$$

In the case of the example 44 rpm right angle drive gear motor and the 8" wheel, this equates to:

$$V_{\text{no-load}} = (\pi \times 8 \text{ in})/\text{rev} \times 44 \text{ rev}/\text{min} = 1105 \text{ in}/\text{min} = 18 \text{ in}/\text{sec} = 1.5 \text{ ft}/\text{sec}$$

Note the 44 RPM speed rating of the motor is under no load. Ideally, we would need to know the torque vs. speed characteristics of the motor to calculate the true shaft speed under the load (i.e. weight) of moving the robot around. **For a first approximation, let's take 75% of the no-load speed rating.** Therefore our robot velocity is now reduced to

$$V = .75 \times V_{\text{no-load}} = 13.5 \text{ in}/\text{sec} = 1.125 \text{ ft}/\text{sec}$$

Referring back to a problem statement from a previous semester, the arena length is 20 feet, the buckets are placed about 2 feet from the end of the arena, the robot passes through a 6 foot long tunnel and the robot starts 5 feet from the tunnel entrance. In addition, the buckets appear to be located about 6 feet apart from each other. Therefore, depending on your bucket collection strategy, you can approximate the distance traveled by the robot. The most direct path for collecting the two buckets would be approximately:

$$d \approx 2 \times (5 \text{ ft} + 6 \text{ ft} + 18 \text{ ft}) + 6 \text{ ft} + 2 \text{ ft (for backing up)} \approx 66 \text{ ft}$$

Now that we know the estimated distance and velocity, it's easy to calculate the estimated driving time (not including the time required to dump, sort and release the buckets):

$$t = d / V = 66 \text{ ft} / 1.125 \text{ ft}/\text{sec} \approx 60 \text{ sec}$$

Before each group selects its final drive motors, we want to see a similar time estimate computation.

As an example of why we want to perform these simple calculations, suppose we ignore the suggestion to read the spec sheet for each motor before selecting one. Instead, we choose one of the Globe DC inline gear motors because of their impressive size and cool looks. Repeating the above calculations:

4.5 RPM 12 VDC GLOBE GEARMOTOR

Double reduction gear motor. Primary gear motor is removable from secondary reduction for use as a 25 RPM output. Ideal motor for robotics, rotary actuators, and other **low speed** DC applications.



SPECIFICATIONS

- RPM: primary 25; secondary **4.5**
- Voltage 12 DC
- Amps 130 mA no load
- Torque: primary 33 in-lb;
secondary 125 in-lb
- Ratio 620:1
- Rotation reversible
- Duty continuous
- Mount primary 4 bolt on 1" BC
secondary 5 bolt on 4 7/8" BC

Notice the RPM specification on the secondary reduction of the motor/gearbox assembly: 4.5 RPM. This is approximately 10% of the speed rating of the first motor we looked at (44 RPM), so the robot velocity will be reduced 90%:

$$V_{\text{no-load, GLOBE}} = (\pi \times 8 \text{ in})/\text{rev} \times 4.5 \text{ rev}/\text{min} \approx 1.8 \text{ in}/\text{sec} = .15 \text{ ft}/\text{sec}$$

$$V_{\text{GLOBE}} = .75 \times V_{\text{no-load, GEARMOTOR}} = 1.35 \text{ in}/\text{sec} = .1125 \text{ ft}/\text{sec}$$

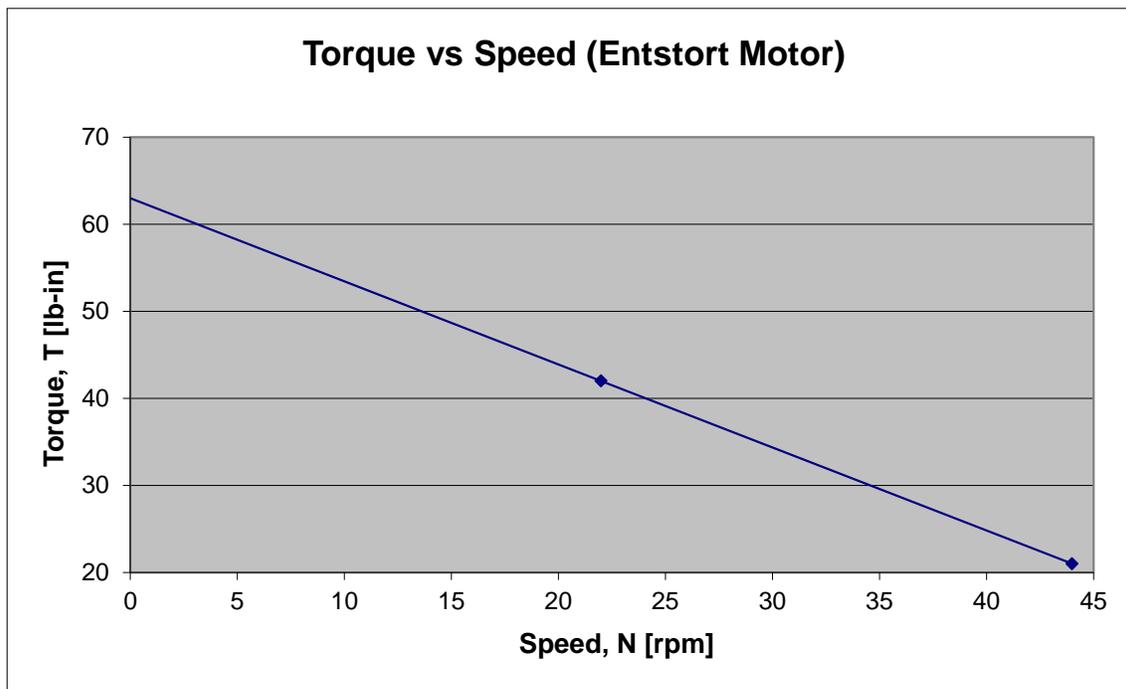
Computing the time required to traverse the arena using the slower Globe gear motor:

$$t = d / V = 66 \text{ ft} / .1125 \text{ ft}/\text{sec} \approx 590 \text{ sec} \approx \mathbf{10 \text{ min (!)}}$$

Characterizing Motors

- Speed (rotational velocity), \mathbf{N} [rev/min] or $\boldsymbol{\omega}$ [rad/s]
- Angular acceleration, $\boldsymbol{\alpha}$ [rad/s²]
- Torque, \mathbf{T} [lb-ft] or [N-m]
 - $T = F \times r$
 - $T = I \times \alpha$
- Power – \mathbf{P} – the rate at which work is done
 - $P = T \times \omega$

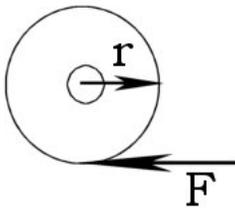
Electric motors are characterized by torque vs. speed curves, such as the following:



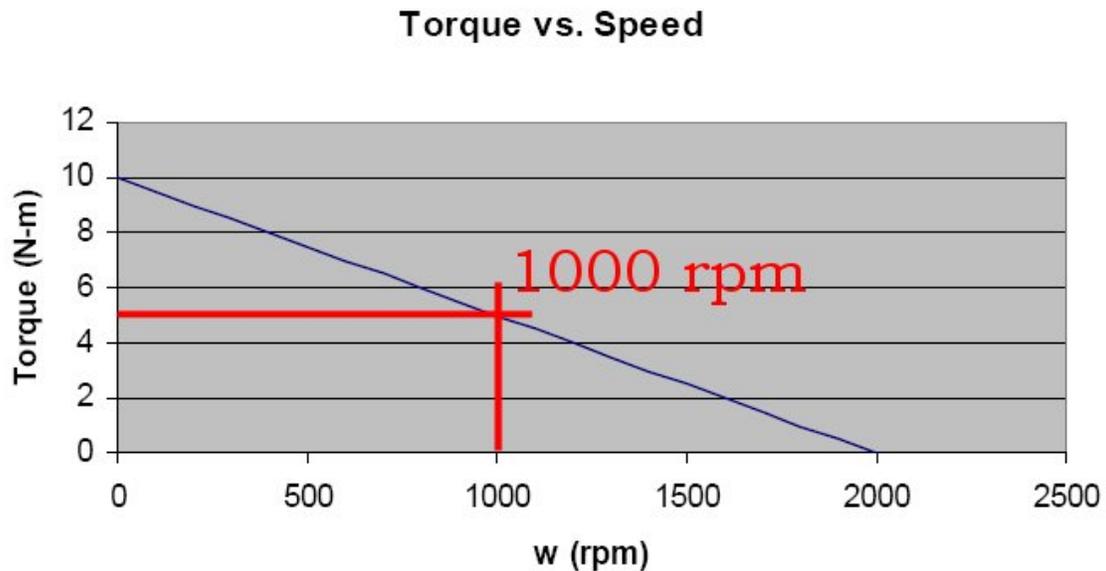
You make use of the torque / speed curve as follows:

1. Calculate or measure the force required to propel the device. For example, let's assume 50 N force.
2. Calculate the torque needed as $T = F \times r$; assuming $r = 0.1$ m:

$$T = 50 \text{ N} \times 0.1 \text{ m} = 5 \text{ N-m}$$



3. Determine motor speed from torque curve.



4. Calculate new wheel speed using the actual motor shaft velocity from the torque vs. speed data ($V = r \times \omega$; $V = \pi \times D \times N$; etc.)

A Closer Look at Calculating the Required Wheel Motor Torque

In the above example, for simplicity, I assumed a force of 50N was necessary to move the robot. Now let's look at how to really calculate this number. To find the actual force, we need to measure or estimate the static friction coefficient between the floor and wheels, since that is what we are overcoming when we push the robot forward (assuming axle friction is negligible). If we look up this static friction coefficient (which is dependent on the two materials in contact, i.e. perhaps a concrete floor and a nylon wheel) and we know the weight on each wheel (W_1 , W_2 , etc), we can calculate the friction force as the static friction coefficient times the wheel weight for each wheel on the robot. If we sum these friction forces, then we know how much force is needed to overcome the static friction to get the robot rolling. By equating torques, you can translate this force into a wheel or a motor shaft torque. That's the basic calculation.

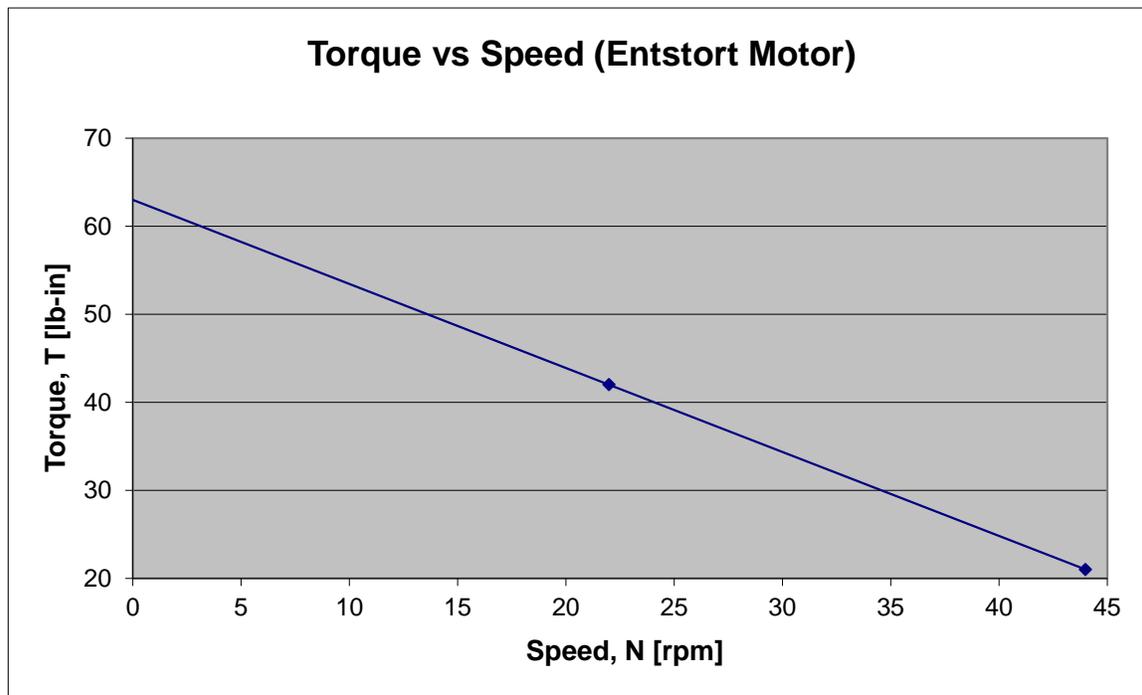
Beyond that, we need to use superposition and add the extra torque needed to accelerate the robot at the intended rate. For instance, if we desire the robot to accelerate at 1 ft/s^2 , we can calculate the equivalent inertia of the robot's weight reflected to the wheels (let's call this I). Once we know this, we can take our intended acceleration, α , and compute the addition torque needed as $T = I \times \alpha$. Summing these two torque values tells us how much overall wheel torque we need to (1) just get the robot moving and (2) to accelerate the robot at the designed rate.

Plotting the Torque vs. Speed Graph for a Real Motor

Now that we know how to calculate the required motor torque, let's look at how to find the actual motor speed. Using the first motor in this lecture as an example, (the Entstort gear-motor), recall the peak motor speed was 44 rpm and we calculated the power as 95 lb-in/s. Since we know the torque vs. speed curve is linear, we need two data points.

We know that power is the rate at which work is performed ($P=T \times \omega$) where omega is the shaft rotational velocity in rad/s. In this case, we computed the power P as 95 lb-in/s. Therefore **$95 \text{ lb-in/s} = T \times 44 \text{ rev/min} \times [2\pi \text{ rad/rev}] \times [1 \text{ min} / 60 \text{ sec}]$** which gives us $T = 20.6 \text{ in-lb}$. This means this motor can lift 20.6 lb at a 1" level arm or radius, or 1 lb at a 20.6" radius, **MAXIMUM**.

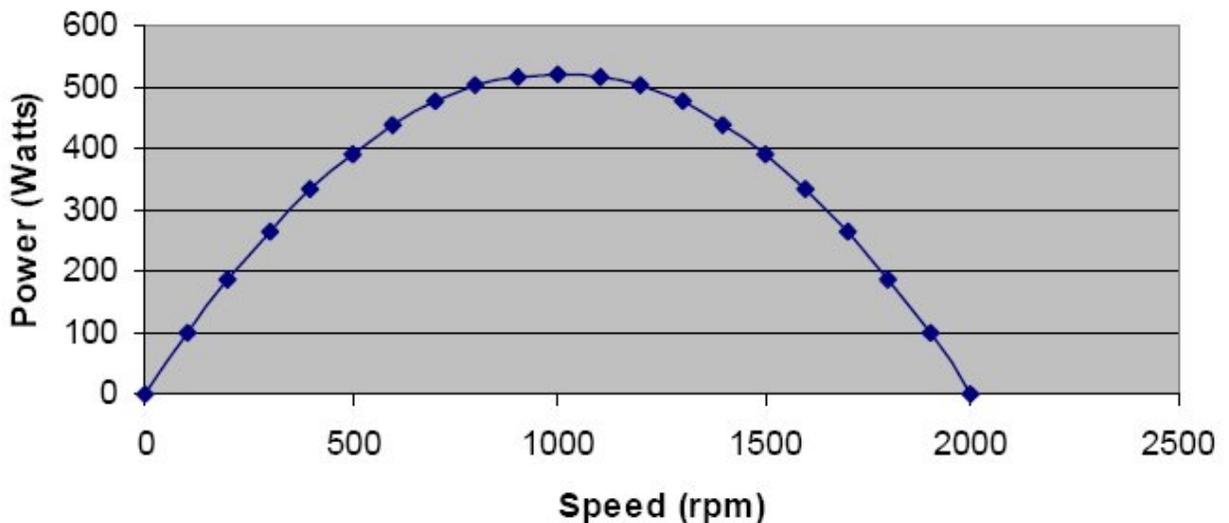
For our second data point, let's use $N = 22 \text{ rpm}$. Since we know the torque vs. speed relation for this motor is linear, we can calculate the motor torque at this speed as equal to 41.2 lb-in. (At half the speed the torque will be twice as much.) Below is the resulting torque vs. speed plot for this motor.



Power Trends for Motors with Linear Torque Curves

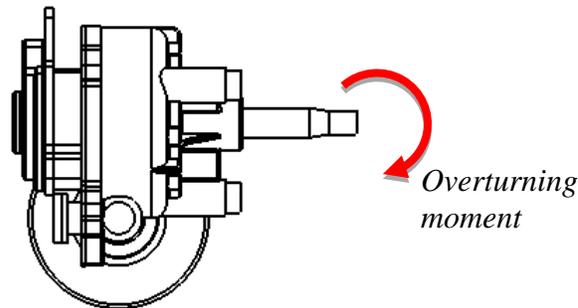
As you can see from the motor torque curve presented in the above example, the motor torque is inversely proportional to motor speed. Since power is equal to torque times motor speed, plotting motor power as a function of speed produces the following graph. **This example shows peak power will always occur at half the *no-load speed* of the motor, when the motor has a linear torque curve.** Therefore, for our initial understanding of permanent magnet DC motors, we can assume that peak motor efficiency also occurs at this speed. Consequently, we want to operate the motor as close to this speed as possible. **If we select the motor for our application based on the required torque, as long as the motor operates between 500 and 1500 rpm, that would be within 25% of the motor's peak efficiency (390W vs. 520W), which would be acceptable.** Operating outside this range results in inefficient power transfer and indicates a design error when selecting motors.

Power vs. Speed



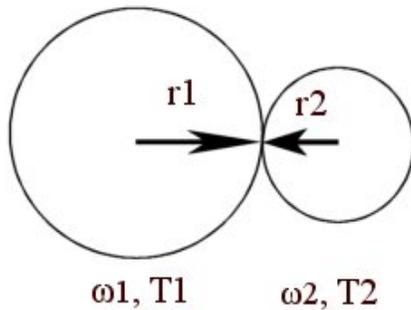
Motor Use Tips

1. **Do not overload motors with excessive overturning moments**, which refer to moments that are not coaxial with the shaft. Most motor gearboxes are not designed to resist large overturning moments, and exceeding the limit will destroy the motors. As a general rule of thumb for the small DC gear motors used in our laboratory, *the overturning moment on the motor shaft should NEVER exceed the rated torque output of the motor. In addition, whenever possible, good design engineers mitigate the adverse effects of overturning moments by using bearings or bushings to support the load applied to the motor shaft.*



2. **Use ALL provided mounting holes when attaching a motor to a mounting bracket.** Most electric motor housings are made from cheap die-cast aluminum or zinc, which are weak metals. Consequently, using more mounting holes/fasteners to attach the motor better distributes the reaction forces to the motor housing. Therefore, motor mounts should always be designed to use all of the motor's mounting holes.
3. **Be very cautious of the fragile motor wires and fastener threads.** The wires entering a motor's casing are easily broken if pulled on or bent tightly, and once broken they cannot be repaired. Similarly, threads in the motor housing are weak to begin with, so it is imperative to use the proper fasteners specified on the motor's spec sheet. A simple rule of thumb to prevent fastener damage is to **always** ensure the mounting fasteners can be screwed together completely by hand before using a tool to apply the final tightening torque.

Gearing



The peripheral velocities at the points of contact between the two gears must be equal, so we write:

$$V_1 = V_2 \Rightarrow \omega_1 r_1 = \omega_2 r_2 \Rightarrow \omega_2 = \omega_1 r_1 / r_2 \quad (\text{Eq. 5})$$

If we know the ratio of gear teeth or diameters, we can easily calculate the relative gear velocities:

$$\text{If } r_1 = 2 r_2 \Rightarrow \omega_2 = 2 \omega_1 \quad (\text{Eq. 6})$$

For any pair of gears, the forces transmitted between the teeth in contact must be equal (and opposite):

$$F_1 = F_2 \Rightarrow T_1 / r_1 = T_2 / r_2 \Rightarrow T_2 = T_1 r_2 / r_1 \quad (\text{Eq. 7})$$

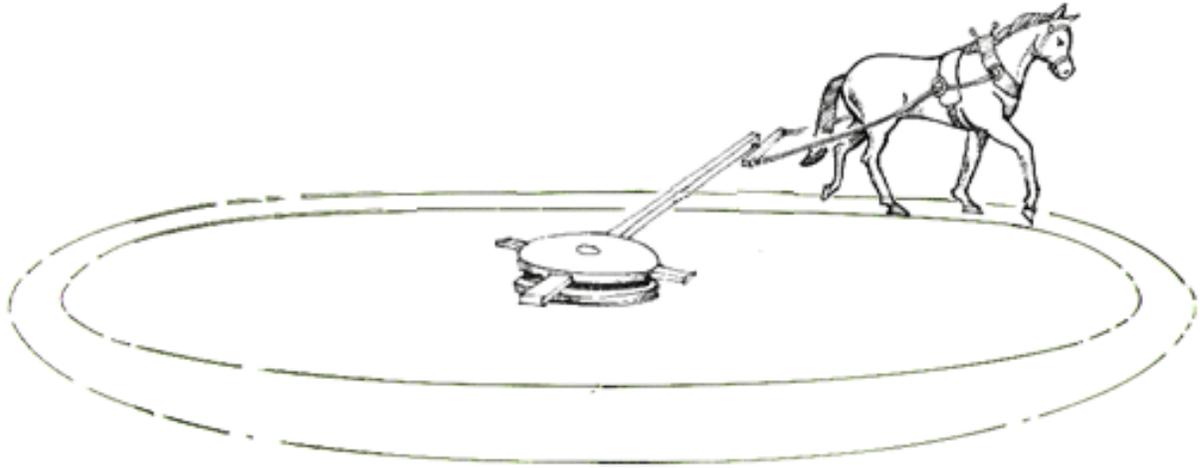
Finally, substituting the fixed tooth or diameter ratio:

$$\text{If } r_1 = 2 r_2 \Rightarrow T_2 = T_1 / 2 \quad (\text{Eq. 8})$$

Which proves the speed of gear 2 is doubled at the expense of gear 2 only transmitting half the torque to its output shaft. Stated another way, gear 1 is capable of transmitting twice the torque thru its output shaft at half the speed of gear 2.

Origins of Horsepower Unit:

In the early 1700's James Watt determined a horse could turn a 12' radius mill wheel 144 times in an hour pulling with a force of 180 pounds



Since we know power is the amount of work performed divided by the time required to perform it:

$$P = W / t = F \times d / t$$

Substituting the data presented at the top of the page from Watt's measurements:

$$1 \text{ hp} = [180 \text{ lb} \times 2\pi \text{ rad/rev} \times 12\text{ft}] / [1 \text{ hr} / 144 \text{ rev} \times (60 \text{ min} / 1 \text{ hr})]$$

Which provides the standard definitions for power:

$$1 \text{ hp} = 33,000 \text{ lb-ft/min} \quad (\text{Eq. 9})$$

$$1 \text{ hp} = 33,000 \text{ lb-ft} / \text{min} \times 1 \text{ min} / 60 \text{ sec} = 550 \text{ lb-ft} / \text{sec} \quad (\text{Eq. 10})$$